CHILD DEVELOPMENT PERSPECTIVES

Links Between the Intuitive Sense of Number and Formal Mathematics Ability

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ABSTRACT—***Humans share with other animals a system for thinking about numbers in an imprecise and intuitive way. The approximate number system (ANS) that underlies this thinking is present throughout the lifespan, is entirely nonverbal, and supports basic numerical computations like comparing, adding, and subtracting quantities. Humans, unlike other animals, also have a system for representing exact numbers. This linguistically mediated system is slowly mastered over the course of many years and provides the basis for most of our formal mathematical thought. A growing body of evidence suggests that the nonverbal ANS and the culturally invented system of exact numbers are fundamentally linked. In this article, we review evidence for this relation, describing how group and individual differences in the ANS correlate with and even predict formal math ability. In this way, we illustrate how a system of ancient core knowledge may serve as a foundation for more complex mathematical thought.***

KEYWORDS—*number; estimation; math*

In many cultures, we esteem those who excel in math and worry about students who struggle. Math ability affects job attainment and success (Rivera-Batiz, 1992), health care choices (Reyna, Nelson, Han, & Dieckmann, 2009), retirement decisions, and salary size (Dougherty, 2003). In addition to the large individual differences in formal math ability that affect the lives of numerate adults, even broader differences exist between those who learn a system of mathematics at all and those who do not (such as some modern human populations; Pica, Lemer, Izard, & Dehaene, 2004). To date, nonhuman species have not demonstrated formal math skills.

Despite these wide differences in formal mathematical ability, a more basic sense of number appears in all typically developing humans and in a range of animals. Infants, including newborns, recognize numerical changes to arrays (even controlling for nonnumerical dimensions such as surface area), compare numbers of items across sensory modalities, and add and subtract approximate quantities (e.g., Feigenson, 2011; Izard, Sann, Spelke, & Streri, 2009; McCrink & Wynn, 2004; Xu & Spelke, 2000). Other nonverbal populations, including rats, fish, monkeys, and birds, also exhibit numerical representations across diverse tasks (Brannon & Merritt, 2011; Feigenson, Dehaene, & Spelke, 2004 for reviews), which suggests that representing imprecise numerical information is likely an evolved, core capacity.

REPRESENTATIONS OF APPROXIMATE NUMBER

Although infants and animals share some numerical competence, this ability appears to depend on a different system of representation than the system that supports formal mathematical abilities in educated children and adults. Infants' and animals' numerical performance (e.g., when judging which of two dot arrays is more numerous) is imprecise, with success dependent on the ratio between quantities (see Brannon & Merritt, 2011; Feigenson et al., 2004, for reviews). For example, 6-month-old infants discriminate 8 from 16 dots, but not 8 from 12 (Xu & Spelke, 2000). The approximate number system (ANS) representations that support this performance can be thought of as a series of Gaussian tuning curves organized on a mental number line: As quantities along the number line

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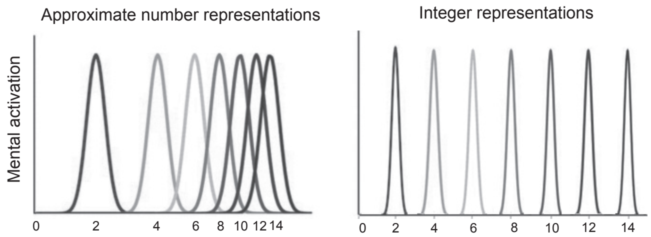

Figure 1. **Models of approximate number representations (left) and exact integer representations (right), depicting distributions of mental activation as a function of target numerosity.**

increase, the curves representing each quantity increasingly overlap, making nearby numbers harder to distinguish (Figure 1).

In contrast, linguistically mediated integer representations are exact, allowing thinkers to recognize that 17 differs from 18 by precisely the same amount that 1017 differs from 1018. This precision makes integer representations quite different from the analog representations of the ANS (Carey, 2009). Indeed, the ANS is too representationally limited to support much of the computation required for the mathematical thinking that many children master by primary school. For example, the ANS cannot represent "exactly 17" as reliably distinct from 16 or 18. And although numerate children and adults do acquire the precise integer representations that support thinking about "exactly 17," they also retain and use the ANS throughout their lives. When choosing the greater of two symbolically presented quantities, adults' speed still depends on the quantities' ratio (Moyer & Landauer, 1967).

The ANS undergoes important developmental change between infancy and adulthood. Specifically, the acuity of its representations sharpens over time (i.e., the Gaussian distributions representing number may narrow and overlap less). This occurs rapidly over the 1st year of life, with infants requiring a 1:2 ratio to discriminate quantities at 6 months, but just a 2:3 ratio at 9 months (Lipton & Spelke, 2003). ANS acuity continues to improve over a protracted period, as shown by experiments in which children and adults see briefly flashed arrays of items and must identify the larger quantity. In these nonverbal, nonsymbolic tasks, the ANS gradually comes to support discriminations of a 6:7 ratio by the time a child is 6 years (Halberda & Feigenson, 2008; Piazza et al., 2010) and continues to sharpen until the individual is about 30 years, after which ANS acuity declines, slowly but steadily, for the rest of life (Halberda, Ly, Wilmer, Naiman, & Germine, 2012).

LINKS BETWEEN ANS AND MATHEMATICAL ABILITY: GROUP DIFFERENCES

How is the ontogenetically and evolutionarily ancient ANS related to the formal mathematical abilities that only some humans acquire? One way to answer this question is to determine whether groups of people known to struggle with formal mathematics also have poorer ANS acuity. If they do, it would suggest that the quality of the representations in one system affects those of the other.

People who have dyscalculia (also known as mathematical learning disability) have serious and persistent difficulty in mathematics despite adequate opportunities for math education and typical learning in other domains. Estimated to affect between 6% and 10% of the population (Butterworth, 2010), dyscalculia may stem from difficulty decoding numeric symbols (e.g., Rousselle & Noël, 2007) or from domain-general deficits in abilities such as working memory and visuospatial processing (Geary, 2004). However, dyscalculic children also show significantly less accurate nonsymbolic numerical approximation than typically developing children of the same age and general cognitive abilities (Andersson & Östergren, 2012; Mazzocco, Feigenson, & Halberda, 2011a; Piazza et al., 2010). In other studies, dyscalculic children were found to be unimpaired in nonsymbolic tasks (De Smedt & Gilmore, 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noël, 2007). This difference in findings may suggest different subtypes of dyscalculia (Geary, 2004; Wilson & Dehaene, 2007). Overall, dyscalculic individuals sometimes show deficits in the representation of quantities—even when neither numerical symbols nor arithmetic computations are required. Hence, dyscalculia may be caused, in part, by impairment of the basic sense of approximate quantity (especially for those who are the most severely mathematically impaired; Mazzocco et al., 2011a).

LINKS BETWEEN ANS AND MATHEMATICAL ABILITY: INDIVIDUAL DIFFERENCES

If individuals who are seriously impaired in formal mathematical abilities also show poorer numerical approximation, is the link between the two systems observable in the general population or does it emerge only when comparing impaired with unimpaired groups? A test of the approximation abilities of typically developing 14-year-olds used psychophysical modeling to estimate the precision of each participant's ANS representations, which was indexed as the Weber fraction, or w (Halberda, Mazzocco, & Feigenson, 2008). The distribution of observed w scores revealed that participants' ANS precision varied surprisingly widely, with some adolescents discriminating quantities differing by a 9:10 ratio and others struggling with quantities differing by a 2:3 ratio. This variability measured at age 14 correlated with variation in formal mathematical ability (measured by the Test of Early Mathematical Ability, 2nd ed. [TEMA–2], and the Woodcock–Johnson revised calculation subtest) across 10 years, administered starting when the participants were in kindergarten. Furthermore, ANS acuity was related to mathematical ability even when controlling for nonnumerical cognitive abilities such as general intelligence, visuospatial ability, and working memory.

This association between ANS acuity and mathematical ability also exists later in life, after formal mathematics training has ended for most people. Adults' ANS precision correlated with their performance on a standardized test of symbolic math ability (the quantitative portions of the SAT or GRE exam), even when controlling for performance on standardized tests of verbal ability (De Wind & Brannon, 2012; Libertus, Odic, & Halberda, 2012). Similarly, adults' ANS acuity correlated with mental arithmetic performance on a speeded arithmetic test (Lyons & Beilock, 2011). This relation remained robust throughout life: Participants' *w* scores correlated with their self-reported ability in school mathematics when controlling for self-reported ability in school science, writing, and computer proficiency at every age examined, from 11 to 85 years (Halberda et al., 2012).

Does the relation between the ANS and formal mathematics depend on having completed many years of mathematics education? Several studies suggest not. For kindergarteners, performance on a nonsymbolic addition task (mentally adding two dot arrays, then comparing the imagined outcome with a third array) predicted general mathematical ability (assessed by tasks involving simple counting and identification of Arabic numerals) 2 months later, controlling for individual differences in verbal ability (Gilmore, McCarthy, & Spelke, 2010). A similar result was found in children as young as 3 years old, whose ANS precision correlated with their performance on a standardized mathematics test (TEMA–3; Libertus, Feigenson, & Halberda, 2011). In addition to the precision of approximate number representations, the mental organization of ANS representations also affects mathematical performance. When producing quantity approximations (pressing a button to rapidly generate, for example, 52 dots on a computer screen) or estimating where a number falls on a number line, children whose estimates of different target quantities were better fit by a linear than a logarithmic function scored higher on standardized mathematics tests (Booth & Siegler, 2006; Sasanguie, De Smedt, Defever, & Reynvoet, 2011). This suggests that approximate number representations may start out as logarithmically organized but become linear as children progress through education in mathematics—with linear representations providing better support for formal mathematics. Thus, by the time children are of preschool age and continuing throughout life, the quality of representations generated by the core system of approximate number is linked to the ability to successfully engage in mathematics.

Although many studies now demonstrate a link between the ANS and mathematical ability, the extent of this relation merits further research. Differences in ANS precision have been found to account for 30% of the variance in mathematical tasks (Halberda et al., 2008), whereas other tasks find a significant but smaller contribution (e.g., 7%; Libertus, Odic, et al., 2012). Other studies have failed to find a relation between the ANS and mathematical ability in typically developing children (Holloway & Ansari, 2009; Iuculano et al., 2008; Soltesz, Szucs, & Szucs, 2010) or have found a significant relation in children but not in adults (Inglis, Attridge, Batchelor, & Gilmore, 2011; see also Castronovo & Göbel, 2012; Price, Palmer, Battista, & Ansari, 2012). Detecting a relation between ANS and mathematical ability may depend on sample size and other characteristics of the subject population, as well as the nature of the tasks used to measure each. For example, some formal mathematical abilities may be more heavily influenced by the ANS than others. However, in a recent study of more than 10,000 participants that used a broad measure of mathematical achievement (self-reported classroom abilities and SAT scores), the relation was modest in size, but highly significant ($r = -.21, p < .000001$; Halberda et al., 2012).

ON THE CAUSAL DIRECTION OF INFLUENCE

The link between the ANS and mathematical ability can be interpreted in several ways. One is that stronger mathematical abilities (perhaps due to differences in the quality and/or quantity of mathematics education) sharpen ANS representations. Alternatively, more precise approximate number representations might make some individuals better at math.

Both interpretations are plausible. First, differences in exposure to or practice in math appear to affect ANS acuity. Although individuals from an innumerate culture that lacks number words and any formal system of mathematics perform well on a numerical approximation task, they are less accurate than educated European adults (Pica et al., 2004). This suggests that receiving sustained mathematics instruction and/or routinely engaging in numerical thought in daily life (e.g., when counting change at a coffee shop or determining the time in New York when vacationing in Alaska) may hone ANS representations.

However, stable individual differences in the ANS are observable even in infancy: Infants who made finer numerical discriminations than their peers at 6 months also made finer numerical discriminations at 9 months (Libertus & Brannon, 2010). These individual differences in the ANS emerged before any opportunity for differences in children's mathematics education or engagement. Furthermore, findings that individual differences in ANS precision *predict* mathematical ability suggest that basic approximation may causally influence formal mathematical ability. Children's ANS precision at age 3 or 4 years (before most children have begun formal mathematics instruction) predicts their standardized mathematics scores at age 5 or 6 years. In contrast, early ANS precision does not predict vocabulary size or the ability to rapidly identify colors or letters (Mazzocco, Feigenson, & Halberda, 2011b). Moreover, individual differences in 4-year-old children's ANS precision predict mathematical ability 6 months later, even when controlling for individual differences in mathematical ability at the time of initial testing (Libertus, Feigenson, & Halberda, 2012; Figure 2). The finding that ANS precision predicts *growth* in mathematical ability likely indicates that ANS representations play some causal role in the acquisition of mathematical ability.

The most powerful evidence of a causal relation between the ANS and mathematical ability would be a demonstration that experimentally manipulating one system affected the other. Very little work of this type exists: Few studies offer evidence that the

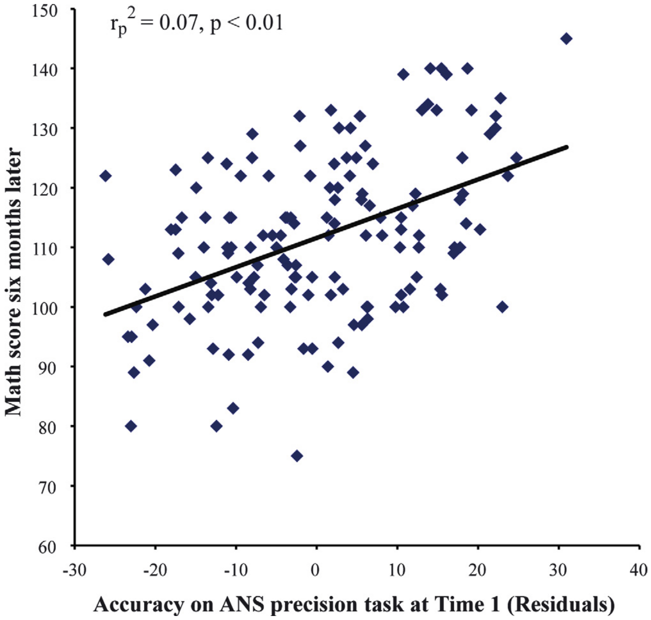

Figure 2. Relation between children's accuracy on a nonsymbolic numerical discrimination task at about age 4 years and their score on a test of symbolic math ability (Test of Early Mathematical Ability–3) 6 months later, adjusted for age at time of testing.

ANS is sensitive to laboratory training, and even fewer suggest that ANS improvement in turn benefits formal mathematical ability. After several weeks of practice on a nonsymbolic approximate addition and subtraction task, adults' ANS precision increased, as did their formal mathematical ability (Park & Brannon, 2012). This result holds promise, but more work remains, including determining the degree of ANS malleability. Among the questions to address: What kind of experience can alter ANS acuity and how much improvement in precision can occur? When adults were given feedback during a nonsymbolic, approximate numerical comparison task, their numerical precision increased quickly (De Wind & Brannon, 2012). However, sustained practice at the task over a 2-week period did not lead to any further benefit. Other studies found that several weeks of practice playing an adaptive numerical comparison task (designed to improve core numerical intuitions) improved some number skills in kindergarteners (e.g., numerical comparison), but not others (e.g., counting or arithmetic; Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009). Further work is needed to assess the replicability of Park and Brannon's (2012) positive finding that training the ANS improves mathematics performance and to determine how robust this effect may be.

HOW ARE ANS AND MATHEMATICS LINKED?

The work reviewed in this article suggests that the ontogenetically and evolutionarily ancient core sense of approximate number is linked to formal mathematical ability. But how should we understand this link? Research has not yet revealed how performing mathematics might improve ANS representations or how ANS representations might improve formal mathematical ability, but several ideas merit consideration.

First, the acquisition and frequent use of number symbols may help sharpen ANS representations (Verguts & Fias, 2004), which is consistent with the finding that numerate adults have more precise ANS representations than adults whose culture lacks number symbols (Pica et al., 2004). However, this account makes additional predictions that should be empirically tested, such as the idea that children should experience a discontinuous jump in ANS precision upon learning the meaning of the number words, at about age 4.

Second, more accurate ANS representations might give children a stronger foothold into the exact integer system early on, when they are first mastering number word meanings. More confidence in numerical meanings might in turn lead to deeper engagement in formal mathematics. This idea predicts that individual differences in ANS representations will correlate not only with mathematical ability but with early number word learning. It also predicts that the period from about 3 to 5 years of age, when children are becoming proficient number word users, is especially ripe for showing effects of training the ANS.

A third possibility is that throughout life, better ANS representations serve as a helpful "check" on symbolic mathematics computations. Sharper ANS representations may allow children and adults to detect gross errors in answers to math problems. It might also be that individual differences in the ordinal representation of approximate quantities, rather than in their cardinal representation, are most critical. Adults' ability to mentally order quantities (e.g., to decide whether symbols were arranged in numerically ascending order) correlated with performance on a symbolic arithmetic task (Lyons & Beilock, 2011). This held true even when controlling for individual differences in ANS precision, which also correlated with mathematics performance. Lyons and Beilock (2011) suggest that understanding the relative meanings of number symbols, which is key for mathematics performance, is grounded in an understanding of the relative meanings of approximate quantities. Having a better sense of the ordered relation between mental magnitudes may help people perform the kinds of computations that are central to formal mathematics.

CONCLUSIONS

People differ greatly in their attitudes toward and proficiency in mathematics. These differences stem in part from environmental factors (e.g., Jordan, Kaplan, Ramineni, & Locuniak, 2009; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Melhuish et al., 2008). In this article, we reviewed evidence suggesting that another critical component to formal mathematical ability is the contribution of an evolved system for representing approximate quantities—one that is shared by human infants, rodents, and fish. Although this idea is still in its early stages, the link between the ANS and mathematics is an exciting area of study because it may help improve mathematics education and provide better support for individuals with math learning disability—although such implications remain to be fully developed. This link is also important as a case study of how core knowledge systems, which make human thinking similar to that of distantly related creatures, may be a basis for formal knowledge systems that are unique to our species.

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