Acta Psychologica 176 (2017) 78–84

Contents lists available at ScienceDirect

Acta Psychologica

journal homepage: www.elsevier.com/locate/actpsy

Approximate number sense correlates with math performance in gifted adolescents

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ARTICLE INFO

Keywords:
Approximate Number System
Symbolic math performance
Individual differences
Talented Youth

ABSTRACT

Nonhuman animals, human infants, and human adults all share an Approximate Number System (ANS) that allows them to imprecisely represent number without counting. Among humans, people differ in the precision of their ANS representations, and these individual differences have been shown to correlate with symbolic mathematics performance in both children and adults. For example, children with specific math impairment (dyscalculia) have notably poor ANS precision. However, it remains unknown whether ANS precision contributes to individual differences only in populations of people with lower or average mathematical abilities, or whether this link also is present in people who excel in math. Here we tested non-symbolic numerical approximation in 13- to 16-year old gifted children enrolled in a program for talented adolescents (the Center for Talented Youth). We found that in this high achieving population, ANS precision significantly correlated with performance on the symbolic math portion of two common standardized tests (SAT and ACT) that typically are administered to much older students. This relationship was robust even when controlling for age, verbal performance, and reaction times in the approximate number task. These results suggest that the Approximate Number System is linked to symbolic math performance even at the top levels of math performance.

1. Introduction

Mathematical thinking permeates modern human life and supports a wide range of activities, from counting change after a purchase to formulating theoretical proofs. Because we rely on mathematics in so many ways, it is not surprising that math competence predicts a variety of long-term outcomes such as job attainment, salary, financial literacy, and personal debt (Dougherty, 2003; Gerardi, Goette, & Meier, 2013; Parsons & Bynner, 2005; Rivera-Batiz, 1992; Roszkowski, Glatzer, & Lombardo, 2015). Individual differences in math ability can be seen starting early in life: whereas some children consistently have difficulty mastering math procedures and concepts (Butterworth, Varma, & Laurillard, 2011; Geary, 2004), other children demonstrate advanced mathematical performance early on (Brody & Mills, 2005).

Many factors have been shown to predict children's math achievement, including family income (e.g., Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006), quality of early childcare (Dearing, McCartney, & Taylor, 2009), teachers' math knowledge (Hill, Rowan, & Ball, 2005), quantity of teachers' math input (Klibanoff et al., 2006), and domain general abilities such as working memory, attention, and executive control (e.g., Clark, Pritchard, & Woodward, 2010; Geary, 2011; Mazzocco & Kover, 2007; Welsh, Nix, Blair, Bierman, and Nelson (2010). But in addition to these factors, an emerging body of research has found that an evolutionarily ancient, non-symbolic sense of quantity is linked with math performance in children and adults. This number sense is seen in human infants starting in the first few days of life (Izard, Sann, Spelke, & Streri, 2009) as well as in a variety of non-human animals including monkeys, rats, chicks, and fish (for review see Agrillo, Piffer, Bisazza, & Butterworth, 2012; Brannon & Merritt, 2011; Feigenson, Dehaene, & Spelke, 2004). While none of these non-verbal creatures represents exact integer quantities, they all can represent quantity in an approximate way. The Approximate Number System (ANS) representations that underlie their performance have been described "noisy," with the amount of noise or uncertainty in the numerical representations scaling with target size (i.e., greater uncertainty for larger values). As a result, observers' ability to numerically discriminate two arrays using the ANS depends on the arrays' ratio rather than their absolute difference (e.g., Halberda & Odic, 2014; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Whalen, Gallistel, & Gelman, 1999). In humans, the noisiness of ANS representations decreases over development, starting in infancy and continuing into adulthood (Halberda & Feigenson, 2008; Lipton & Spelke, 2003; Xu, Spelke, & Goddard, 2005), with humans' highest precision attained at around 30 years of age (Halberda, Ly, Wilmer, Naiman, & Germine,

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http://dx.doi.org/10.1016/j.actpsy.2017.03.014
Received 23 May 2016; Received in revised form 27 March 2017; Accepted 29 March 2017
Available online 04 April 2017

2012).

Noisy ANS representations are fundamentally different from the precise integer representations that play a key role in human mathematical reasoning. For example, discriminating the nearby quantities 49 and 50 using the ANS is difficult (and would yield performance that is only just above chance in humans (Halberda, 2016)), but discriminating 49 from 50 using integer representations is easy. Whereas approximate number representations are used by humans starting in infancy, integer representations are not exhibited until around age four, after the verbal counting procedure is acquired (Carey, 2009; Wynn, 1992). Whereas no human culture has been documented to lack approximate number representations, adult humans in cultures that lack number words appear to lack integer representations (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). And whereas a wide range of animal species represents approximate number, no non-human species has shown evidence of representing integers.[1]

Yet despite these differences between approximate number representations and the integer representations on which so much of formal mathematics depends, evidence suggests a link between the two. In particular, individual differences in ANS precision have been found to correlate with symbolic math performance in multiple age groups (Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013 for a review). In such studies, ANS precision typically is measured by showing observers two stimulus arrays (e.g., dot displays) and testing their ability to reliably judge which is more numerous. Although this nonverbal, non-symbolic task is trivially easy when the ratio between the arrays is large (e.g., 20 dots versus 40 dots), it becomes much harder when the ratio is small (e.g., 38 dots versus 40 dots). Critically, people differ in their precision at this task; some observers can reliably discriminate even very small ratios, but others require larger numerical differences between arrays in order to succeed. In adults and adolescents, individual differences in this simple, non-symbolic measure of ANS precision retrospectively correlate with scores on standardized math tests taken many years earlier, with various non-numerical factors controlled for (Halberda, Mazzocco, & Feigenson, 2008; Halberda et al., 2012; Libertus, Odic, & Halberda, 2012). In children, ANS precision correlates with concurrently measured symbolic math performance (Bonny & Lourenco, 2013; Guillaume, Nys, & Mussolin, 2013; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Feigenson, & Halberda, 2011; Lourenco, Bonny, Fernandez, & Rao, 2012; Odic et al., 2016). And in preschoolers, ANS precision measured at one time point (even as early as infancy) predicts future math performance (Chu & Geary, 2015; Libertus, Feigenson, & Halberda, 2013; Starr, Libertus, & Brannon, 2013; van Marle, Chu, Li, & Geary, 2014). Taken together, these findings suggest that the primitive, non-verbal sense of approximate number is linked with symbolic math from early in life.

However, not all examinations of the relationship between the ANS and symbolic math have found evidence of a correlation (Iuculano, Tang, Hall, & Butterworth, 2008; Price, Palmer, Battista, & Ansari, 2012; Sasanguie, DeSmedt, Defever, & Reynvoet, 2012). And for some of the cases in which a correlation was found, researchers have suggested that ancillary factors, rather than any direct link between the ANS and math performance, might be responsible. For example, Gilmore et al. (2013) suggested that individual differences in the ability to inhibit non-numerical dimensions when comparing arrays may underlie the relationship between the ANS and math performance (also Fuhs & McNeil, 2013, but for counter-evidence see Keller & Libertus, 2015). Others have argued that correlations between ANS precision and math are caused by individual differences in lower level visuospatial abilities (Tibber et al., 2013), although this does not explain the observed correlation between ANS precision (for sequences of tones) and math performance in congenitally blind individuals (Kanjlia, Feigenson, & Bedny, submitted). Still, although the relationship between the ANS and symbolic math abilities remains an active area of inquiry, meta-analyses suggest that, across the full body of studies of the ANS and math, there is a reliable relationship between the two (r = 0.24, 95% CI [0.14 0.26], Chen & Li, 2014; r = 0.24, 95% CI [0.20 0.28], Schneider et al., 2016).

Moreover, this link between ANS and symbolic math seems to go beyond the correlational. Emerging evidence finds that training observers in approximate number comparisons tasks not only improves their ANS precision, but also improves symbolic math performance (Hyde, Khanum, & Spelke, 2014; Park, Bermudez, Roberts, & Brannon, 2016; Park & Brannon, 2013; Wang, Odic, Halberda, & Feigenson, 2016). For example, we recently used a training task designed to either temporarily enhance or impair the ANS precision of 5-year old children. These children showed respective benefits or impairments in a subsequent symbolic math task, and no change in a subsequent verbal task.

But is the ANS linked with symbolic math ability in everyone? ANS representations and symbolic math ability have been found to correlate in children struggling with dyscalculia – a math-specific deficit (Landerl, Fusseneger, Mill, & Willburger, 2009; Mazzocco, Feigenson, & Halberda, 2011; Mejias, Grégoire, & Noël, 2012; Olsson, Ostergren, & Träff, 2016; Piazza et al., 2010; Skagerlund & Traff, 2016; but see also Iuculano et al., 2008). Many other studies have found a link in participants who exhibit symbolic math performance in the typical ability range (Anobile, Castaldi, Turi, Tinelli, & Burr, 2016; Bonny & Lourenco, 2013; Chu & Geary, 2015; Guillaume et al., 2013; Halberda et al., 2012; Halberda et al., 2008; Inglis et al., 2011; Libertus et al., 2012; Libertus et al., 2013; Lourenco et al., 2012; Odic et al., 2016; van Marle et al., 2014). But notably, studies of mathematically gifted individuals are missing from this picture.[2] How could such studies contribute to our understanding of the link between ANS precision and symbolic mathematics? Consider the widespread view that the ANS is a "primitive" system (i.e., a system that supports intuitive number thoughts in infants, and across animals and human cultures). This view may lead to theories of the causal role of the ANS in symbolic mathematics that focus on the early school years and on very basic math abilities (e.g., ordinal comparison); these theories might also predict that gifted students performing high-level mathematics will not show a link between their math performance and the ANS. For instance, one might expect that ANS representations will not play a major role in our understanding of and computing of binomial functions or cubed roots. In contrast, if the role of the ANS in symbolic math abilities extends beyond the early years and beyond 'primitive' operations, then we may observe a link between ANS performance and symbolic math abilities even in gifted students.

Here we asked whether individual differences in ANS precision are linked to math in high achieving individuals by studying adolescents enrolled in the Center for Talented Youth (CTY) program at the Johns Hopkins University. Each year, top 7th- and 8th-grade students (< 5% of the population) are invited by CTY to apply to their summer enrichment program. These students typically are initially selected because they performed above the 95th percentile in nationally normed tests or were recommended by teachers on the basis of exceptional academic records. The students are then screened using standardized college entrance exams such as the SAT or the ACT, which are usually administered when children are in 12th grade. Students qualify by achieving the required minimum scores on these tests – about the 50th

[1] Even for animals trained to map numerical representations to symbols (Boysen & Berntson, 1989; Matsuzawa, 1985; Pepperberg, 1994), their performance suggests that they do not exhibit integer representations. Rather, these animals may have learned mappings between the ANS and digits or specific vocalizations.

[2] Although Mazzocco et al. (2011) found that adolescents identified as high achieving in math (> 95th percentile) exhibited better ANS precision than typically achieving children, this difference did not reach significance in their sample. However, their study focused on children at the lower end of the math performance distribution, and may have been underpowered to examine the relationship between ANS precision and math performance in high achievement.

percentile (among the select subgroup of talented 7th and 8th graders who take the tests early) in either the math or verbal portion of the exam, or the combined score (CTY, 2016). These CTY students are at least several years away from graduating from high school when they take these tests, but generally perform as well or better than average high school seniors bound for college (Assouline & Lupkowski-Shoplike, 1997). Top performance by 7th and 8th grade children on such standardized tests has been found to predict later achievements, including the ranking of the college later attended, college grade point average, and obtaining a graduate school degree (Benbow, 1992).

We tested a cohort of these high achieving students in a non-symbolic numerical comparison task that allowed us to assess their ANS precision. We also accessed the standard test scores (either SAT or ACT) that students had used to apply to the CTY program. We hypothesized that students' ANS precision would correlate with their math test scores even when controlling for their age and verbal test scores.

2. Methods

2.1. Participants

Twenty-one students (mean age = 15.07 years, *SD* = 1.03; range = 13.32 to 16.91 years; 10 females) enrolled in the Center for Talented Youth (CTY) summer school at Johns Hopkins University participated in exchange for monetary compensation. All participants' parents provided written permission prior to testing. Participants provided informed written assent. All participants had normal or corrected-to-normal visual acuity, and none reported color blindness. One additional participant was excluded because his ANS precision was more than three standard deviations from the sample average, suggesting inattention during the task. No participants performed more than three standard deviations from the mean on either the Math or Verbal SAT/ACT tests, and hence no students were excluded on the basis of their standardized test scores.

2.2. ANS comparison task

To measure the precision of each student's Approximate Number System (ANS) we administered a version of Panamath (Psychophysical Assessment of Numerical Approximation; www.panamath.org)–a non-symbolic numerical comparison task that requires participants to rapidly judge the relative numerosity of two dot arrays, with varying array numerical ratios across trials. Each participant was tested individually in a quiet dimly lit room. All displays were presented on a Macintosh iMac computer at a viewing distance of approximately 60 cm, with the display subtending $39.56^\circ \times 25.35^\circ$ of visual angle.

Participants saw a series of arrays containing two spatially separated collections of blue and yellow dots on a grey background, and were asked to indicate whether more of the dots were blue or yellow. The numerical ratio between the arrays was calculated by dividing the larger number of dots by the smaller. The ratios varied randomly from the hardest trials, with a ratio of 1.11 (18:20 dots) to the easiest trials, with a ratio of 2.5 (6:15 dots). There were between 5 and 22 dots in each collection (blue and yellow). The ratios were categorized into 4 ratio bins: 1.1, 1.3, 1.4, and 2.4, with 24 trials in each ratio bin, yielding a total of 98 trials. The color of the more numerous array varied randomly across trials. To discourage participants from relying on the cumulative area of dots, on half of the trials the numerically larger collection was also larger in cumulative surface area (Surface Area Congruent trials), on the other half of the trials the two collections were equated in cumulative surface area (Surface Area Equated trials). In addition, individual dot size varied within each array. On each trial the dots in one array had an average diameter of 36 pixels (range: 16–56 pixels); dot size in the other array was calculated based on condition (i.e., on Surface Area Congruent trials the dots in the other array also averaged 36 pixels in diameter, and on Surface Area Equated trials average dot diameter was determined by the numerical ratio being presented).

Participants first saw a white fixation cross on the grey background, after which they pressed the spacebar to initiate each trial. The two stimulus arrays were presented simultaneously for 800 ms (too briefly to allow the dots to be serially counted), followed by a blank grey screen. Yellow dots always appeared on the left. Participants were instructed to respond using the keyboard, pressing the "F" key if there were more yellow dots and the "J" key if there were more blue dots. They heard a high-pitched tone immediately following their key press when they were correct, and a low-pitched tone when they were incorrect. The task lasted about 5 min.

2.3. Standardized math and verbal scores

Participants' parents received a consent form in which they gave permission to have their child's test scores (SAT or ACT) provided to the experimenters by CTY. On the day of the experiment, students gave written assent to release their scores.

The math subtests of both the SAT and ACT measure performance in a range of math topics. These include aspects of algebra, such as solving and graphing linear equations (e.g., multiple choice problem: "Which of the following numbers is not a solution of the inequality $3x - 5 \geq 4x - 3$?"), solving quadratic equations, performing operations with polynomials, and interpreting nonlinear expressions (e.g., multiple choice problem: "If $(ax + 2)(bx + 7) = 15cx^2 + cx + 14$ for all values of x, and $a + b = 8$, what are the two possible values for c?"), problem solving and data analysis (including interpreting data tables and graphs), and geometry and trigonometry (e.g., "In a right triangle, one angle measures x°, where $\sin x^\circ = \frac{4}{5}$. What is $\cos(90^\circ - x^\circ)$?") (the College Board and ACT.org). The verbal portion of the SAT and ACT (now called the "Reading section") measures a range of reading skills, including the ability to find evidence in a passage, use context to understand meaning of words (e.g., after reading a passage, multiple choice problem: "In line 88 of the passage, "adhere" most nearly means which of the following?"), and draw inferences based on a passage (e.g., after reading a passage, multiple choice problem: "The author of this passage would most likely agree with which of the following statements about the man referred to in line 2?") (the College Board and ACT.org).

Most test questions were multiple-choice (except for a subset of questions on the SAT Math test), and students were time constrained (SAT-Math: 70 min; SAT-Reading: 70 min; ACT-Math: 60 min; ACT-Reading: 35 min). All students took either the SAT or ACT during the years 2010–2013, when they were in the 7th or 8th grade; students' age averaged 13.36 years at the time of SAT or ACT testing.

3. Results

Each participant's Math and Verbal SAT or ACT scores were converted into percentiles based on the national test profile report of college-bound 12th grade students for the year the test was taken (College Board, 2010–2013; ACT.org, 2010–2012); this allowed us to analyze across SAT and ACT scores despite their different scales. Each participant received a Math and Verbal percentile score that reflected their performance relative to the same-year cohort of 12th grade students taking the same test. The average Math percentile in our sample was 77.19 ($SD = 19.52$) and the average Verbal percentile was 74.14 ($SD = 19.27$). Both of these were significantly above the 50th percentile of 12th grade students, as revealed by one-sample t-tests on Verbal percentile, $t(20) = 5.74$, $p < 0.001$, $d = 1.39$, and Math percentile, $t(20) = 6.38$, $p < 0.001$, $d = 1.25$. This suggests that the students in our sample, who were in the 7th and 8th grade at the time they took the tests, performed better than average college-bound high school students. There was no significant difference between Verbal and Math performance in our sample, $t(20) = 0.55$, $p = 0.59$, Cohen's

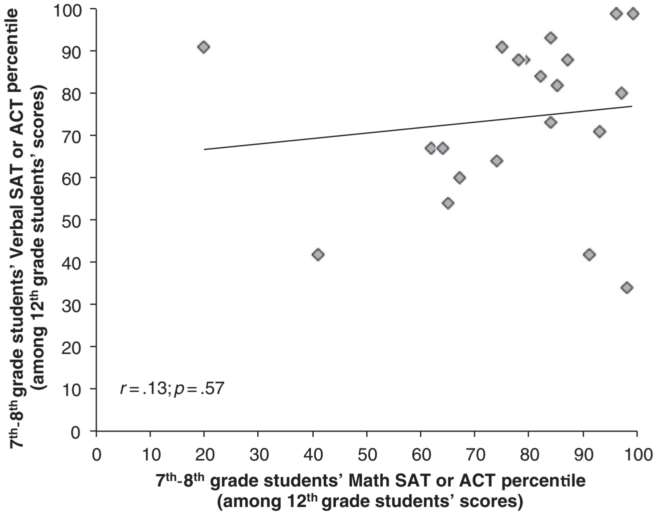

Fig. 1. Math SAT or ACT performance in relationship to Verbal SAT or ACT performance (p = n.s.).

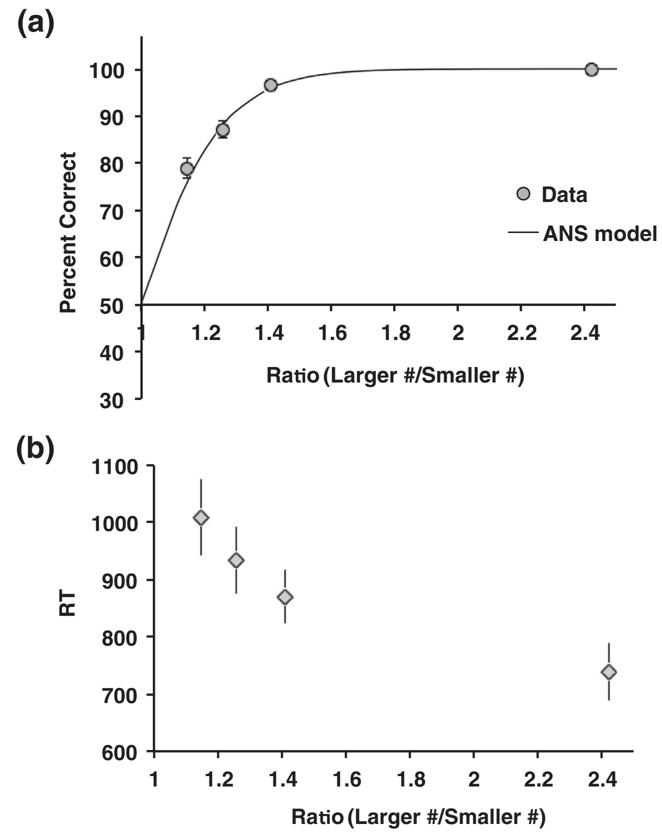

Fig. 2. (1) Percent correct on ANS Comparison test as a function of numerical ratio. Line reflects fit of ANS model to group performance; all analyses relied on fitting the model to each participant's performance. (2) Response time (RT) on ANS Comparison test as a function of numerical ratio. Bars plot standard error of the mean.

$d = 0.12$, and no significant correlation between Verbal and Math performance, $r = 0.13$; $p = 0.57$ (see Fig. 1). We also did not find any significant difference between the performance of males and females on either the Math, $t(19) = 0.33$, $p = 0.75$, Cohen's $d = 0.14$, or Verbal subtests, $t(19) = -1\ 0.69$, $p = 0.11$, Cohen's $d = -0.74$.

On the ANS comparison test, participants responded correctly on 89.48% of trials (SD = 4.27%) and their average response time was 857.94 ms (SD = 243.95 ms). Consistent with the psychophysical signature of the Approximate Number System, they were more accurate and faster on trials with larger numerical ratios (Fig. 2). A repeated-measures ANOVA with Ratio as the independent variable and accuracy as the dependent measure revealed a significant effect, $F(3,56) = 50.71$, $p < 0.001$, $\eta_p^2 = 0.72$ (all ANOVA results Greenhouse-Geisser corrected for non-sphericity). Similarly, a repeated-measures ANOVA revealed a significant effect of Ratio on RT, $F(3,44) = 19.09$, $p < 0.001$, $\eta_p^2 = 0.51$. Also consistent with the psychophysical signature of the ANS, there was a significant correlation between accuracy and response time, $r = 0.69$, $p = 0.001$, suggesting that students who were more precise at ANS discrimination were faster at making ANS-based decisions – note that this is the opposite direction of a speed-accuracy tradeoff. Students performed better on Surface Area Congruent trials (92.13% correct) than Surface Area Equated trials (89.31% correct). A repeated-measures ANOVA with Area Congruency (Congruent vs. Equated) as the independent variable and accuracy as the dependent measure revealed a significant effect, $F(1,20) = 5.24$, $p = 0.03$, $\eta_p^2 = 0.21$. This expected effect reveals that, while visual size did affect responses, all participants were able to respond based on numerosity, as revealed in above-chance performance for both Area Congruent, $t(20) = 46.56$, $p < 0.001$, $d = 10.15$, and Surface Area Equated trials, $t(20) = 31.03$, $p < 0.001$, $d = 6.78$.

To find each individual participant's Weber fraction (w), an estimate of their ANS precision, we fit each participant's response over all 98 trials with a commonly-used psychophysical model (Halberda & Feigenson, 2008; Halberda et al., 2008; Pica et al., 2004). In this model, the representations of the two different numbers on each trial are modeled as Gaussian random variables with means n_1 and n_2 and standard deviations equal to w multiplied by the respective mean. Accuracy is modeled as 1 minus the error rate, which is defined as the area under the tail of the resulting Gaussian: $\frac{1}{2}\,erfc\left(\frac{n_1 - n_2}{\sqrt{2}\,w\sqrt{n_1^2 + n_2^2}}\right)$. This model implies that as the ratio of two numbers becomes closer to 1, their Gaussian representations should tend to overlap more and it should be more difficult for participants to determine which array has more dots, resulting in decreased accuracy. There is only one free parameter w, which reflects the amount of noise in the underlying ANS representations. Larger w values indicate larger standard deviations and therefore poorer ANS precision. The best fitting w value for each participant was determined by finding the least-squares fit on their average accuracy for each ratio bin. Individual w values ranged from 0.02 to 0.21 and average $w = 0.13$ ($SD = 0.05$).

The critical question motivating this work was whether ANS precision was correlated with symbolic math performance in high achieving children. Since students were of different ages when they took the standardized math and verbal tests, we first performed preliminary analyses on the effect of age at time of testing. We did not find any significant correlation between the age at which students took the SAT/ACT and their Math, $r = 0.29$, $p = 0.20$, or Verbal performance, $r = -0.004$, $p = 0.99$. Nor did we observe a significant correlation between students' age when they took the ANS comparison test and their w, $r = -0.11$, $p = 0.63$. Although previous studies find that ANS precision is related to age (Halberda et al., 2008; Halberda et al., 2012), the lack of age effect is likely due to the small age range in our sample.

A linear regression of w with Math percentile revealed a significant correlation, $r = -0.46$; $p = 0.03$ (Fig. 3a). This correlation was comparable across the two types of ANS Comparison trials (Surface Area Incongruent trials vs. Surface Area Equated trials), Fisher's test $p = 0.76$. The correlation between accuracy on the Surface Area Incongruent trials and Math percentile was $r = 0.42$, $p = 0.06$; and the correlation between accuracy on the Surface Area Equated trials and Math percentile was $r = 0.50$, $p = 0.02$. In contrast, w did not correlate with Verbal percentile, $r = -0.03$, $p = 0.93$ (Fig. 3b). Similar relationships were observed when we used accuracy instead of w to index ANS precision. Accuracy on the ANS Comparison task significantly correlated with Math percentile, $r = 0.54$, $p = 0.01$, but not

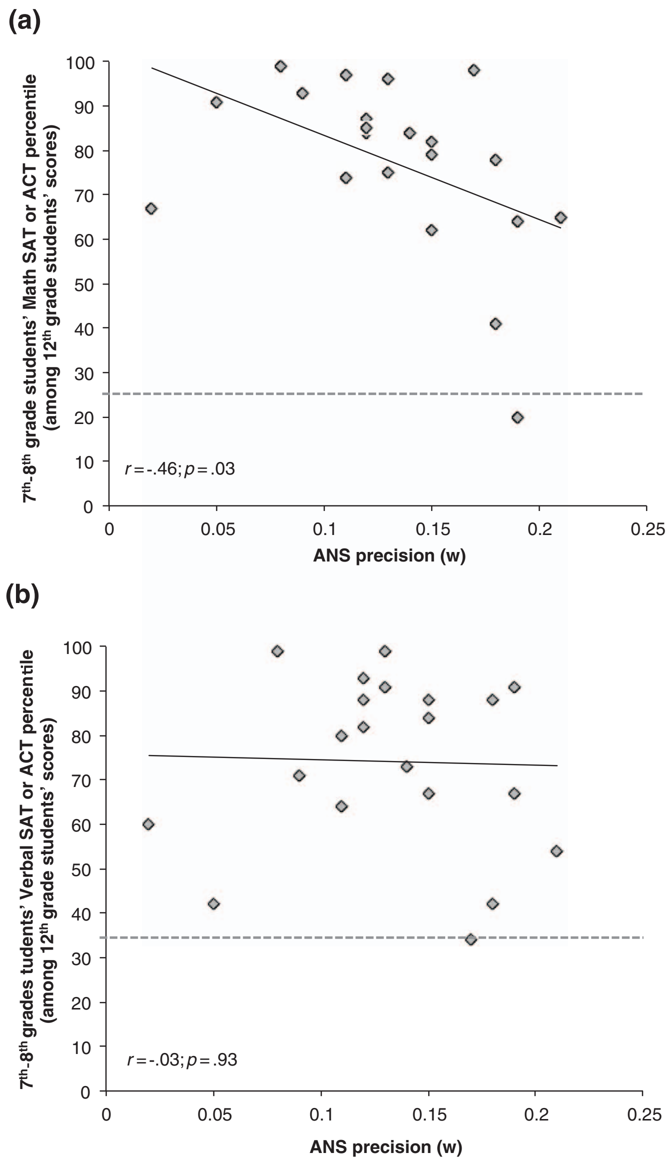

Fig. 3. ANS precision in relation to SAT or ACT performance. (a) ANS precision (w) correlated with Math scores (SAT, ACT) ($p < 0.05$). (b) ANS precision (w) correlated with Verbal scores (SAT, ACT) (p = n.s.). Percentiles reflect performance of the 7th and 8th grade students in our sample, compared to the same-year performance of college-bound 12th grade students who took the tests nationally. Dashed lines represent estimates of the average SAT/ACT performance of the academically advanced 7th grade students who participated in Talent Search programs (College Board, 2012).

Verbal percentile $r = 0.12$, $p = 0.62$.

In order to control for differences in students' age when they took the SAT/ACT, age when they took the ANS Comparison test, and Verbal ability, we ran an additional model using the residuals of Math percentile, w, and RT, while partialling out age at time of taking the SAT/ACT, age at time of taking the ANS Comparison test, and Verbal percentile. A regression model predicting Math-residuals using w-residuals and RT-residuals was marginally significant, $r^2 = 0.23$, $F(2,18) = 2.62$, $p = 0.10$ (Table 1). The beta score between Math-residuals and w-residuals was significant, suggesting a reliable correlation between Math percentile and w when controlling for students' age when they took the SAT/ACT, age when they took the ANS comparison task, Verbal percentile, and response time (Table 1). The beta score between Math-percentile and RT was in the predicted direction, but did not reach significance (Table 1). As noted earlier, superior ANS performance can be seen in both smaller w scores (i.e., less noise in the underlying ANS representations) and smaller RTs (i.e., less time required to use the ANS representations to make a decision). The beta scores for both w and RT were negative, showing that children who were faster and more precise on the ANS task did better in formal math when controlling for age and Verbal percentile.

Table 1
Linear regression analyses predicting Math percentile using response time (RT) and ANS precision (w) while partialling out students' age when taking the SAT/ACT, age when taking the ANS comparison test, and Verbal percentile.

Predictor	*Beta*	r_p	t	p
w-Residuals	− 0.64	− 0.24	− 2.19	0.04
RT-residuals	− 0.30	− 0.46	− 1.05	0.31

4. Discussion

Previous research has found that individual differences in the precision of the evolutionarily ancient, non-symbolic Approximate Number System correlate with math achievement (for review see Chen & Li, 2014; Feigenson et al., 2013). However, this past work was limited to individuals who either exhibited average math performance, or who specifically struggled with math. Our results extend these findings by showing that the precision of approximate number representations is linked to math performance even at the highest levels of math achievement. Here we found that among adolescents enrolled in an exclusive program for gifted students, nearly all of whom were performing math at least several grade levels ahead, those who were better at discriminating non-symbolic numerical quantities also showed better performance in the math subtest of standard college admission tests. Crucially, this result obtained when accounting for age, response time, and performance in the verbal portion of the same admission test.

The mechanism by which ANS precision is associated with symbolic math abilities remains unknown. One hypothesis is that the ANS plays an instrumental role in acquiring elementary numerical skills, with poorer precision in ANS representations leading to difficulties in, for example, acquiring exact meanings of integers (Mundy & Gilmore, 2009). These early emerging numerical difficulties might result in decreased engagement in math learning, which in turn could lead to struggles with school mathematics later in life. Equally, better ANS precision early in childhood could result in earlier or deeper engagement with basic symbolic math concepts early on; this early math advantage might then lead to acceleration of the mastery of later, higher level math concepts. On such an account, better ANS precision is not directly linked to superior performance in advanced mathematics, but instead benefits simpler math skills that are acquired early; better basic math skills lead to better advanced math skills.

A second hypothesis is that the ANS is associated with more sophisticated mathematical thinking as well as with basic numerical processing. Having a more precise ANS may help students better understand and engage even higher-level mathematical concepts. Recent work finds that when professional mathematicians judge the truth value of advanced mathematical statements (involving algebra, geometry, and topology, without using any number words), they activate the classic bilateral network of prefrontal, parietal, and inferior temporal brain regions that is often seen during number processing. Notably, circuits typically involved in language and general semantic processing were not specifically recruited (Amalric & Dehaene, 2016). This suggests that even at the highest levels of math achievement—in professional mathematicians—quantitative thinking draws in part on a very basic brain network for numerical thought. This, along with our present findings, raises the possibility that individual differences in the processing performed by this network could directly contribute to high level math achievement.

Finally, our data are also consistent with the possibility that the link between the ANS and math reflects causal influence in the opposite direction: that greater quantity and quality of engagement in mathematical activities results in increased ANS precision among high math achievers (see Piazza, Pica, Izard, Spelke, & Dehaene, 2013; Shusterman, Slusser, Halberda, & Odic, 2016). These three possibilities are, of course, not mutually exclusive.

One source of evidence relevant for adjudicating the above hypotheses is students' performance on the individual math problems comprising the SAT and ACT. Although all of the problems required symbolic math understanding, it is possible that the correlation between ANS precision and math performance was driven by the subset of problems that involved numerical magnitudes (such as solving algebraic equations), as opposed to problems more distant from magnitudes (such as factoring polynomials). Unfortunately, our data cannot provide such evidence, as we only had access to students' composite scores on the Math and Verbal portions of the SAT and ACT. Future research examining different aspects of symbolic math performance and its relationship with ANS precision will be valuable for characterizing the nature of the role ANS plays in symbolic math performance. At present, our findings suggest that examining individual differences in a very basic cognitive system–one that we share with other animals and that humans deploy across development–can shed light on the mathematical successes of even highly gifted children.

Acknowledgements

This research was conducted at Johns Hopkins University and was funded by National Institutes of Health grant R01 HD057258. The authors declare no conflict of interest that might influence the research. The authors gratefully thank all the participants of this study. We thank Stuart Gluck and Amy Shelton at the Center for Talented Youth at Johns Hopkins University for coordinating with us and supporting our research, and we thank Darko Odic, Lauren Quast, and David Strasshofer for helping with data collection.

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