Journal of Experimental Child Psychology 147 (2016) 82–99

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Changing the precision of preschoolers' approximate number system representations changes their symbolic math performance

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A R T I C L E I N F O

Article history:
Received 30 September 2015
Revised 31 January 2016
Available online 8 April 2016

Keywords:
Numerical cognition
Children
Approximate number system (ANS)
Causal relationship
Symbolic math
Confidence hysteresis

A B S T R A C T

From early in life, humans have access to an approximate number system (ANS) that supports an intuitive sense of numerical quantity. Previous work in both children and adults suggests that individual differences in the precision of ANS representations correlate with symbolic math performance. However, this work has been almost entirely correlational in nature. Here we tested for a causal link between ANS precision and symbolic math performance by asking whether a temporary modulation of ANS precision changes symbolic math performance. First, we replicated a recent finding that 5-year-old children make more precise ANS discriminations when starting with easier trials and gradually progressing to harder ones, compared with the reverse. Next, we show that this brief modulation of ANS precision influenced children's performance on a subsequent symbolic math task but not a vocabulary task. In a supplemental experiment, we present evidence that children who performed ANS discriminations in a random trial order showed intermediate performance on both the ANS task and the symbolic math task, compared with children who made ordered discriminations. Thus, our results point to a specific causal link from the ANS to symbolic math performance.

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http://dx.doi.org/10.1016/j.jecp.2016.03.002
0022-0965/

Introduction

Increasing mathematical competence is a central goal of formal education. Yet individuals vary widely in their skill at solving math problems and their comprehension of math concepts. Whereas some children experience great difficulty in mastering mathematical procedures and concepts throughout their lives (Butterworth, Varma, & Laurillard, 2011; Geary, 2004), other children consistently exhibit advanced achievement in mathematics (Brody & Mills, 2005). Given the contribution of mathematics ability to job attainment, salary, and personal debt (Dougherty, 2003; Gerardi, Goette, & Meier, 2013; Parsons & Bynner, 2005; Rivera-Batiz, 1992), there is a pressing need to better understand the sources of individual variability in mathematics comprehension and performance.

One source of variability in mathematical competence is long-term experience with formal mathematics. Differences in the quality and quantity of children's early math learning opportunities have been shown to affect their subsequent math performance (e.g., Beilock, Gunderson, Ramirez, & Levine, 2010; Hill, Rowan, & Ball, 2005; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015).

But in addition to the math-related experiences that parents and teachers provide, children also are influenced by an intuitive, non-symbolic, approximate sense of number that is available prior to the onset of schooling (Izard, Sann, Spelke, & Streri, 2009; Lipton & Spelke, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005) and that remains active across the lifespan (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza, Pinel, Le Bihan, & Dehaene, 2007). This number sense is supported by the approximate number system (ANS; for reviews, see Brannon & Merritt, 2011; Dehaene, 1997; Dehaene & Brannon, 2011; Feigenson, Dehaene, & Spelke, 2004; Halberda & Odic, 2014; Nieder & Dehaene, 2009), which is functional in newborn infants (Izard et al., 2009), is used by adults lacking formal math education (Frank, Everett, Fedorenko, & Gibson, 2008; Pica, Lemer, Izard, & Dehaene, 2004), operates across sensory modalities (Barth, Kanwisher, & Spelke, 2003; Feigenson, 2011; Izard et al., 2009; Libertus, Feigenson, & Halberda, 2013b; Nieder, 2012), and has been demonstrated in a wide range of non-human species (Cantlon, Platt, & Brannon, 2009; Dehaene, Dehaene-Lambertz, & Cohen, 1998). The ANS represents numbers in a noisy imprecise fashion, with the imprecision of its numerical representations growing with the target numerosity. Consequently, the ability to nonverbally numerically discriminate two arrays depends on the arrays' ratio rather than their absolute difference (Halberda & Odic, 2014; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Whalen, Gallistel, & Gelman, 1999). For example, discriminating 8 versus 16 dots (a ratio of 2.0) is as easy as discriminating 20 versus 40 dots and is easier than discriminating 32 versus 40 dots (a ratio of 1.25); in this sense, the ANS produces noisy representations that are unlike the exact integers that form the basis of much of the symbolic mathematics that children encounter.

An emerging body of research suggests that, despite the differences between approximate number representations and the exact, symbolically mediated numbers used in school mathematics, the ANS and symbolic math performance are likely related (for reviews, see Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013; Schneider et al., in press). Evidence in support of this relationship comes from findings that individual differences in ANS precision often correlate with mathematics achievement in children and adults. Performance on standardized math tests has been found to correlate with future ANS ability (Halberda, Mazzocco, & Feigenson, 2008; Libertus, Odic, & Halberda, 2012) and current ANS ability (Bonny & Lourenco, 2013; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Feigenson, & Halberda, 2011; Linsen, Verschaffel, Reynvoet, & De Smedt, 2014; Lourenco, Bonny, Fernandez, & Rao, 2012; Lyons & Beilock, 2011; Odic et al., 2016), and ANS performance predicts future math ability (Gilmore, McCarthy, & Spelke, 2010; Libertus, Feigenson, & Halberda, 2013a; Mazzocco, Feigenson, & Halberda, 2011b; Starr, Libertus, & Brannon, 2013; van Marle, Chu, Li, & Geary, 2014). In addition, children with mathematical learning disabilities (MLDs or dyscalculia) have significantly poorer ANS precision than typically developing children (Brankaer, Ghesquière, & De Smedt, 2014; Mazzocco, Feigenson, & Halberda, 2011a; Piazza et al., 2010; Skagerlund & Träff, 2016), whereas children with high math achievement show superior ANS precision (Wang & Feigenson, in preparation).

Alongside these positive findings of a link between the ANS and math ability, some researchers have failed to find evidence of a correlation between the two (Iuculano, Tang, Hall, & Butterworth, 2008; Price, Palmer, Battista, & Ansari, 2012; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Soltész, Szucs, & Szucs, 2010). Furthermore, when considering the nature of the relationship between the ANS and formal math ability, some researchers have suggested that the observed correlations reflect ancillary factors rather than any direct link between the ANS and math performance. For example, Gilmore and colleagues (2013) found that math achievement correlated with children's ANS precision only on trials where approximate number and area were incongruent. They suggested that the relationship between ANS performance and math performance actually reflects individual differences in the ability to inhibit non-numerical dimensions when making numerical discriminations rather than a fundamental link between different numerical systems (Fuhs & McNeil, 2013; but see also Keller & Libertus, 2015). Others have suggested that correlations between ANS precision and math are driven by individual differences in lower level visuospatial abilities (Tibber et al., 2013). Hence, determining the strength, stability, and source of the relationship between the ANS and symbolic math is an area of active investigation (De Smedt, Noël, Gilmore, & Ansari, 2013; Feigenson et al., 2013; Göbel, Watson, Lervåg, & Hulme, 2014; Holloway & Ansari, 2009).

One important limitation in this endeavor is that existing research has been largely correlational. Although many of the studies on the relationship between the ANS and symbolic math controlled for non-numerical factors, including general IQ and working memory, the causal arrows underlying any potential link remain unclear. Having more precise ANS representations could causally benefit math abilities, or having better math abilities or greater engagement with math might sharpen ANS representations (see Nys et al., 2013; Piazza, Pica, Izard, Spelke, & Dehaene, 2013). It is also possible that the two systems mutually influence each other or that a third factor influences both (Fuhs & McNeil, 2013; Gilmore et al., 2013).

Ideally, to test the hypothesis that ANS precision causally affects symbolic math ability, researchers would experimentally manipulate ANS precision and then measure the effect this has on math performance. Three recent studies provide some evidence that ANS-based training can improve arithmetic performance. In two of these, Park and Brannon (2013, 2014) trained adults to add or subtract two non-symbolic dot arrays and compare the estimated answer with a third dot array. After 10 training sessions, adults' accuracy had significantly improved. Critically, this intervention also appeared to improve performance in a subsequent timed symbolic addition and subtraction task but not in a verbal task. However, the approximate arithmetic practice did not improve adults' precision in pure non-symbolic quantity comparison (judging which of two arrays was greater without addition or subtraction), suggesting that the observed math improvement was likely due to strengthening arithmetic computation (i.e., approximate arithmetic boosting symbolic arithmetic) rather than changes in the precision of the ANS representations themselves (Park & Brannon, 2014). In a third study, elementary school children were randomly assigned to either an ANS-based numerical training task or a non-numerical training task. Children's ANS precision was not changed by either of these types of practice. However, children who received the approximate number training were significantly faster (although no more accurate) at solving subsequent symbolic arithmetic problems than children who had practiced the non-numerical task (Hyde, Khanum, & Spelke, 2014).

These previous studies begin to identify one aspect of the relationship between the ANS and math in showing that practicing tasks that involve the ANS (i.e., approximate addition, subtraction, and comparison) can improve symbolic arithmetic. However, because neither children nor adults showed changes in ANS precision after the training, it remains unknown whether ANS representations themselves can causally affect math performance. That is, it has yet to be shown that training experiences can increase ANS precision and that these changes in precision in turn affect symbolic math. In the current study, we sought to address this question. First, we asked whether temporarily changing ANS precision would change symbolic math. Second, we relied on a new type of manipulation, *confidence hysteresis*, which has been shown to be capable of either enhancing or impairing children's ANS precision compared with a control condition (Odic, Hock, & Halberda, 2014). Unlike previous training studies, this manipulation has the added control benefit of giving all children equal amounts of numerical practice prior to the math task. Finally, we tested children's symbolic math abilities with a wider range of problem types in order to ask whether ANS training affects aspects of mathematical thinking other than arithmetic.

We used a task that has recently been shown to change children's observed ANS precision through a brief training experience. Odic and colleagues (2014) presented 4- to 6-year-olds with a non-symbolic ANS discrimination task (judging which of two dot arrays was more numerous). Critically, they manipulated the order in which children performed easy versus difficult numerical discriminations. Children in the Easy-First condition were tested on a sequence of trials that began with easy discriminations (involving highly discriminable numerical ratios such as 2.0 and 1.5) and gradually progressed to harder discriminations (involving ratios such as 1.14 and 1.11). These children exhibited significantly better ANS precision than children in the Hard-First condition, who completed the identical trials but in the opposite order, progressing from harder to easier ratios. Additional control conditions showed that this performance difference was not due to general factors such as loss of interest, amount of practice, and poor evaluation in the Hard-First condition. For example, when children in the Easy-First condition were provided with reversed feedback (i.e., consistently receiving negative feedback on trials for which they had responded correctly, and vice versa), they still performed better than children in the Hard-First condition despite having received more negative feedback overall (Odic et al., 2014). This suggests that simply receiving positive or negative feedback throughout the task did not change children's performance. However, children required *some* form of feedback to exhibit a change in ANS precision (i.e., either veridical feedback or reversed feedback; Odic et al., 2014). Therefore, it seems that experiencing a growing sense of confidence in their numerical judgments (motivated by feedback-driven attunement to the correctness of their own responses) changed children's numerical precision. The authors termed this effect ANS *confidence hysteresis* in order to highlight connections to literatures in dynamical psychophysics (Hock & Schoner, 2010; see also Ahissar & Hochstein, 1997).

In the current study, we used ANS confidence hysteresis to study the relationship between ANS representations and symbolic math performance in 5-year-old children who had recently begun learning symbolic mathematics. First, in Experiment 1, we presented children with a series of non-symbolic numerical discriminations that progressed either from easier discriminations to harder ones or from harder discriminations to easier ones (following Odic et al., 2014). After this, we tested children's performance on either a symbolic math task or a vocabulary task. If the ANS plays a causal role in symbolic mathematics, briefly changing children's ANS precision should significantly modulate subsequent symbolic math performance but should have no effect on vocabulary performance. Then, in a Supplemental Experiment, we added a control condition in which a new group of children completed the identical ANS discrimination trials from Experiment 1, this time in a random order, and then completed the same symbolic math task. We asked whether this random trial order would result in performance on the ANS discrimination task and the symbolic math task that was intermediate between the Easy-First and Hard-First children.

Experiment 1

Method

Participants

In total, 40 native English-speaking children with a mean age of 5 years 4 months participated (*SD* = 2.15 months, range = 60.30–67.30 months; 16 girls). Of this sample, 20 children were assigned to the Easy-First Manipulation condition and 20 were assigned to the Hard-First Manipulation condition. The sample size was based on a power analysis using the results of Odic and colleagues (2014) to estimate the likelihood of observing a significant difference in the ANS precision of children assigned to the two groups. In this analysis, an *N* of 40 was sufficient to yield an alpha level of .05, two-tailed, with a power of .80 to detect an effect of ANS manipulation condition. Following the ANS task, half of the children in each manipulation condition were tested on a symbolic math task and half were tested on a non-numerical vocabulary task. Thus, we used a 2×2 experimental design with 10 participants per cell.

All participants were healthy children with no reported developmental delays. Parents of all children (100%) reported English as the primary language spoken at home, and 87.5% of parents reported having college degrees or higher. In terms of race/ethnicity, 87% of the participants were White, 5% were African American, 2.5% were Asian, 2.5% were Native Hawaiian, and 7.5% reported being of mixed race/ethnicity or of a racial/ethnic group other than the above (participants could indicate membership in multiple groups). An additional 5 children (1 from the Easy-First Manipulation condition and 4 from the Hard-First Manipulation condition) were excluded based on their failure to correctly answer more than one of the first three questions (i.e., the easiest three questions) on either the symbolic math task (2 children) or the vocabulary task (3 children). Children received a small gift (e.g., T-shirt, book, toy) to thank them for their participation, and parents provided written informed consent prior to the experiment.

ANS Manipulation Task

The ANS Manipulation Task was a replication of that used by Odic and colleagues (2014) to test children of the same age as those tested here. As such, the specific task parameters we used were based on those in their study. Children were individually tested in a sound-attenuated room. They sat approximately 40 cm away from a 13-inch Apple MacBook laptop on which stimuli were presented using a custom-made Java program.

Children were told that they were going to play a game in which they would see dots flashed on a laptop screen and should indicate whether more of the dots were yellow or blue. The screen contained two black frames, each with a cartoon character's image beside it (Big Bird or Grover; see Fig. 1). On each test trial, yellow dots appeared in the left frame (Big Bird's box) and blue dots appeared simultaneously in the right frame (Grover's box). Dots remained visible for 1200 ms, after which they disappeared, leaving only the empty frames and the Big Bird and Grover characters (in our previous work on the ANS, and in the work by Odic and colleagues (2014), it was found that a 1200-ms display time is sufficiently long for children in this age range to comfortably view the stimulus image but sufficiently brief to prevent serially counting the dots). Children indicated their response either by pointing to or naming the color of the more numerous array, and the experimenter immediately pressed the corresponding key on the keyboard (*f* for yellow or *j* for blue), with the computer recording all responses. Children received feedback after every trial; a pre-recorded voice said, "That's right" following correct responses and "Oh, that's not right" following incorrect ones. This feedback was included because, as described above, Odic and colleagues found that presenting trials in increasing or decreasing order of difficulty with no feedback did not shift children's ANS precision.

Our task began with four practice trials of moderate difficulty in which the two dot arrays first appeared sequentially, briefly disappeared, and then appeared again simultaneously for 1200 ms. The experimenter prompted children to say whether there were more yellow dots or blue dots, and children received feedback as in the test trials. Children averaged 78.75% correct on these four practice

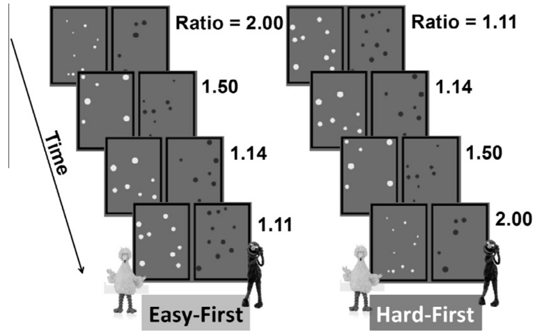

Fig. 1. Example stimuli seen by children in the Easy-First and Hard-First conditions of the ANS Manipulation Task in all of the experiments.

trials, which was well above chance, $t(39) = 6.76$, $p < .001$, and performance on these practice trials did not differ between the two ANS manipulation conditions, $F(1, 38) = 0.34$, $p = .56$, suggesting that all children understood the task.

The four practice trials were followed by 30 test trials in which the arrays were always presented simultaneously for 1200 ms. Across trials, the numerical ratio between the arrays varied across trials among the following: 1.11 (i.e., 10 dots vs. 9 dots), 1.14 (8 vs. 7), 1.17 (14 vs. 12), 1.25 (10 vs. 8), 1.50 (9 vs. 6), and 2.00 (10 vs. 5) (modeled on previous work in which similar numbers of ratios were presented across trials; Libertus et al., 2011, 2013a, 2013b; Odic, Libertus et al., 2013). Each ratio was presented five times with different dot sizes and configurations. We presented a total of 30 ANS manipulation trials because Odic and colleagues (2014) found evidence that a similar number of trials was sufficient to induce changes in children's ANS performance. The side with the larger number of dots was counterbalanced across trials. To discourage children from relying on cumulative area, on half of the trials the array with the larger number also had more cumulative area (congruent trials), and on the other half of the trials the array with the larger number had less cumulative area (incongruent trials).

Children were tested in a pre-generated trial order. In the Easy-First condition, children began with the easiest numerical ratios (e.g., 2.00 and 1.50) and gradually progressed toward the most difficult ratios (e.g., 1.14 and 1.11) (see Appendix A). For children in the Hard-First condition, this sequence was reversed; children began with the hardest numerical ratios and ended with the easiest ratios. All other aspects of the task and displays (e.g., spatial position of individual dots, side of presentation) were identical between the two conditions. The task took approximately 5 min.

Symbolic Math Transfer Task

For half of the children in Experiment 1, immediately following the ANS Manipulation Task, we measured symbolic math performance using 18 items from Form A of the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003). The TEMA-3 is a standardized test of symbolic math ability that has been normed for children between 3 and 8 years of age. It contains 72 items divided into subcategories of informal symbolic math abilities (40 items) and formal symbolic math abilities (32 items). Informal symbolic items test skills that children typically acquire without formal school instruction, including verbal counting and solving spoken math problems using fingers or tokens. In contrast, formal symbolic items on the TEMA-3 test abilities that usually require explicit instruction such as writing Arabic numerals and understanding place values. Each item category in the TEMA-3 was constructed using previous research on children's mathematical abilities (e.g., Baroody, 1995; McCloskey, Caramazza, & Basili, 1985); overall, the TEMA-3 has been shown to have high content validity (Ginsburg & Baroody, 2003). Although the authors of the TEMA have not published factor analyses of TEMA items, subsequent factor analyses found that children's performance was better fit by a model containing multiple subtypes of mathematical abilities, as opposed to a single math ability score (Ryoo et al., 2015).

We presented 18 selected items (rather than administering the whole TEMA-3) because previous work found that ANS precision correlates with all of the subcategories of items in the informal symbolic math category but correlates only with the numeral literacy items in the formal symbolic math category (Libertus et al., 2013b). Therefore, we chose 18 items from the informal subcategories (numbering, number comparison, and calculation) and the numerical literacy subcategory that were within the expected ability range of typically developing 5-year-old children (see Appendix B). These subcategories capture different types of mathematical skills. Numbering assesses verbal counting abilities (e.g., "Count backward starting from 10"). Number comparison involves judging the relative magnitude of spoken number words (e.g., "Which is more, 7 or 6?"). Calculation requires children to solve word problems using external tokens or mental reasoning (e.g., "Joey has 1 block and gets 2 more; how many does he have altogether?"). Numeral literacy involves reading and writing Arabic numerals (e.g., "Can you write the number 7?"). We administered all 18 items, rather than following the TEMA procedure of staircasing each child (i.e., continuing until children err on five consecutive items), and administered them to all children in the same order. This guaranteed that all of the children were tested on the same symbolic math items. As per the TEMA-3 testing manual, children were given only neutral positive feedback throughout (e.g., "Okay! Shall we try another?").

Vocabulary Transfer Task

To test the specificity of any observed effects of ANS manipulation condition on children's subsequent abilities, we tested half of the children in Experiment 1 on a vocabulary task immediately following the ANS Manipulation Task. We chose a vocabulary task as our control for the following reasons. First, there are no documented correlations between ANS precision and verbal knowledge. Hence, if the ANS confidence hysteresis effect is specific to numerical thinking, we predicted that we would see no change in verbal ability between the two training conditions (similarly, verbal knowledge was used as a control task by Park & Brannon, 2013, 2014, and Hyde et al., 2014). Second, the existence of vocabulary tests that, like the TEMA-3, have been normed for preschool- and early elementary school-aged children (e.g., Peabody Picture Vocabulary Test; Dunn & Dunn, 2007) made it possible to administer a vocabulary task of roughly equal difficulty to that of our Symbolic Math Transfer Task, thereby eliminating the possibility of failing to observe transfer due to floor or ceiling performance.

To measure children's vocabulary, we administered 24 selected items from the Peabody Picture Vocabulary Test (PPVT-4; Dunn & Dunn, 2007), which consists of a picture book with four colored drawings per page. For each page, the experimenter reads a word aloud (e.g., "farm", "catching", "digital") and children point to the picture that best matches the word's meaning. The PPVT-4 is normed for English speakers from 2 years 6 months of age through adulthood (81+ years). To make the task similar to the Symbolic Math Transfer Task, we selected 24 items that were predicted to fall within the range of vocabulary knowledge of 5-year-old children. As in the Symbolic Math Transfer Task, we presented the vocabulary task items in the same sequence for all children and provided only neutral positive feedback throughout.

Neither our Symbolic Math Transfer Task nor our Vocabulary Transfer Task should be viewed as administrations of normed instruments (the full TEMA-3 and the full PPVT-4). Instead, our transfer tasks were designed as experimental tools derived from longer, well-studied measures previously administered to children in our age range.

Results

We focused on total percentage correct as a measure of performance for all three of our tasks: the ANS Manipulation Task, the Symbolic Math Transfer Task, and the Vocabulary Transfer Task (for evidence that accuracy reliably indexes the precision of the ANS, see Inglis & Gilmore, 2014). We provide additional analyses of these tasks in our Supplemental Experiment.

We first analyzed children's ANS performance—that is, their accuracy at judging whether there were more blue or yellow dots. Collapsing across the Easy-First and Hard-First conditions (and across transfer tasks), we found no significant differences in children's performance on trials where number was congruent versus incongruent with cumulative area, $F(1, 39) = 0.006$, $p = .94$; therefore, we collapsed these trial types for further analyses.

Next, we tested for an effect of ANS confidence hysteresis. A 2 (Condition: Easy-First or Hard-First) × 2 (Transfer Task: Symbolic Math or Vocabulary) repeated measures analysis of variance (ANOVA) with ANS accuracy as the dependent variable revealed a significant effect of condition, $F(1, 36) = 13.78$, $p = .001$, $\eta_p^2 = .28$, and no effect of transfer task, $F(1, 36) = 0.03$, $p = .86$, or Condition × Transfer Task interaction, $F(1, 36) = 0.11$, $p = .74$. Averaging across two transfer task groups, children in the Easy-First condition responded correctly on 76.84% ($SD = 8.02$) of the ANS discrimination trials, which was significantly better than the performance of children in the Hard-First condition (66.80%, $SD = 8.91$) (Fig. 2). This suggests that we were successful in shifting (either enhancing or impairing) children's ANS performance using a simple 5-min numerical discrimination task.

We next asked whether changing the observed precision of ANS representations affected children's subsequent performance on the Symbolic Math or Vocabulary control task. A univariate ANOVA with ANS manipulation condition (Easy-First or Hard-First) and transfer task (Symbolic Math or Vocabulary) as factors, and with accuracy on the Symbolic Math Transfer Task or the Vocabulary Transfer Task as the dependent variable, revealed no main effect of transfer task, $F(1, 36) = 0.54$, $p = .47$, suggesting that we were successful in matching the overall difficulty of the Symbolic Math Transfer Task and the Vocabulary Transfer Task. Critically, there was a main effect of ANS manipulation condition,

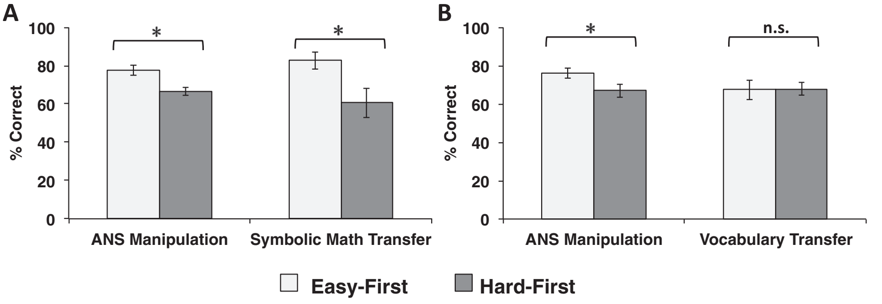

Fig. 2. (A) Children's performance in the ANS Manipulation (training) Task and subsequent performance in the Symbolic Math Transfer Task, separated by ANS manipulation condition (Easy-First or Hard-First). (B) Children's performance in the ANS Manipulation (training) Task and subsequent performance in the Vocabulary Transfer Task, separated by ANS manipulation condition (Easy-First or Hard-First). Error bars indicate ±1 standard error. Asterisks indicate a significant difference between the means, $p < .05$.

$F(1, 36) = 4.12$, $p = .05$, $\eta_p^2 = .10$, and a significant ANS Manipulation Condition × Transfer Task interaction, $F(1, 36) = 4.44$, $p = .04$, $\eta_p^2 = .11$. Children who performed the ANS discriminations in the Easy-First order performed better on the Symbolic Math Transfer Task than children who performed the very same ANS discriminations in the Hard-First order (Easy-First: 82.78%, $SD = 14.21$; Hard-First: 60.56%, $SD = 24.21$) (Fig. 2A). In contrast, children's performance in the Vocabulary Transfer Task was nearly identical across the two ANS manipulation conditions (Easy-First: 67.50%, $SD = 16.17$; Hard-First: 67.91%, $SD = 10.21$) (Fig. 2B). This implies that ANS confidence hysteresis had a specific effect on symbolic math performance.

Discussion

In Experiment 1, we found that the precision of children's ANS discriminations can be altered via confidence hysteresis. Critically, we found that these changes transferred to children's performance on a symbolic math task but did not transfer to a vocabulary task. This provides evidence for a causal link from ANS precision to symbolic math performance and provides some rough boundaries on the scope of this influence (i.e., transfer to a numerical task but no transfer to a language task).

An open question is whether the ANS confidence hysteresis effect improved the performance of children in the Easy-First Manipulation condition, impaired the performance of children in the Hard-First Manipulation condition, or both. Similarly, it remains unknown whether children's math performance in Experiment 1 was improved, disrupted, or both. Therefore, in a Supplemental Experiment, we asked whether presenting children with the same ANS discriminations as in Experiment 1, but this time in a random trial order, would result in performance that was intermediate between the Easy-First and Hard-First Manipulation conditions of Experiment 1. We also analyzed in greater detail children's performance during the ANS Manipulation Task (i.e., as a function of numerical ratio) and performance on the Symbolic Math Transfer Task (i.e., as a function of subcategory of math question).

Supplemental Experiment

Method

Participants

In this experiment, 10 new native English-speaking children with a mean age of 5 years 5 months ($SD = 1.67$ months) were tested in a Random Order ANS Discrimination Task, followed by the Symbolic Math Transfer Task. We chose a sample size of 10 because in Experiment 1 10 children completed the ANS trials in the Easy-First order (and then completed the symbolic math task) and 10 children completed them in the Hard-First order (and then completed the symbolic math task). Therefore, having

10 children complete the same ANS trials in a random order allowed us to compare the groups' performance. Parents of all children (100%) reported English as the primary language spoken at home, and 90% of parents reported having college degrees or higher. In terms of race/ethnicity, 90% of the participants were White and 10% reported being of mixed race/ethnicity. Children received a small gift (e.g., T-shirt, book, toy) to thank them for their participation, and parents provided written informed consent prior to the experiment.

ANS manipulation

The ANS Manipulation Task was identical to that in Experiment 1 except that children completed a trial sequence in which difficulty (i.e., ratio between the two stimulus quantities) was randomly intermixed. The numerical ratios, spatial arrangement of the arrays, controls for non-numerical variables, display timing, and feedback were all identical to those in Experiment 1. Children averaged 70.45% correct on the four practice trials, which was well above chance, $t(9) = 3.45$, $p = .002$. Combined with the results from Experiment 1, children in the Easy-First, Hard-First, and Random Order conditions did not differ in their performance on the practice trials, $F(48) = 0.864$, $p = .357$.

Symbolic Math Transfer Task

The Symbolic Math Transfer Task was identical to that in Experiment 1.

Results

We examined the performance of children who experienced the Random Order ANS manipulation, relative to the children in the Easy-First and Hard-First conditions of Experiment 1. In addition, we examined in greater detail the performance of children in both Experiment 1 and the Supplemental Experiment during the ANS Manipulation Task (as a function of ratio) and the Symbolic Math Transfer Task (as a function of subcategory of math questions). Although the analyses reported below consider the performance of the 20 children from Experiment 1 who were tested on the symbolic math task as a comparison group with the new group of 10 children tested in the Supplemental Experiment, the results are, to a large extent, not redundant with those in Experiment 1 given that those analyses focused on overall percentage correct, whereas below we model children's performance to estimate Weber fractions. We note that given the relatively small sample sizes, our analyses of the subcategories of math skills are intended to be exploratory. All repeated measures analyses are corrected for non-sphericity.

First, collapsing across the Easy-First, Hard-First, and Random Order conditions, we found no significant differences in children's performance on trials where cumulative area was congruent versus incongruent with number, $F(1, 18) = 0.91$, $p = .35$, $\eta_p^2 = .05$, and therefore we collapsed across these two trial types for further analyses.

Next, we analyzed children's performance on the ANS Manipulation Task in terms of percentage correct across the different numerical ratios presented (recall that, despite the differences in trial order, all children were tested with identical ratios). We performed a 6 (Numerical Ratio) $\times$ 3 (Condition: Easy-First, Hard-First, or Random Order) repeated measures ANOVA with ANS accuracy as the dependent measure. This revealed a significant effect of ratio, $F(5, 135) = 13.91$, $p < .001$, $\eta_p^2 = .34$, with children performing better on easier ratios, consistent with the psychophysical signature of the ANS. Critically, we also observed a significant effect of condition, $F(2, 27) = 3.82$, $p = .035$, $\eta_p^2 = .22$, and a marginal interaction between condition and ratio, $F(10, 135) = 1.85$, $p = .079$, $\eta_p^2 = .12$. Tukey's HSD (honestly significant difference) post hoc analyses showed that children in the Easy-First condition who had later been tested on the symbolic math task ($M = 77.55\%$, $SD = 8.09$) performed significantly better than children in the Hard-First condition who had later been tested on the symbolic math task ($M = 66.60\%$, $SD = 7.28\%$), $p < .05$; children in the Random Order condition showed intermediate performance ($M = 72.33\%$, $SD = 11.00$) but were not significantly different from either the Easy-First or Hard-First condition. This provides suggestive evidence that the ANS confidence hysteresis effect can both enhance (Easy-First condition) and disrupt (Hard-First condition) ANS performance.

Using standard psychophysical model and fitting methods (e.g., Odic et al., 2014), we explored the group fits that provided an estimate of the observed ANS precision for each group (i.e., group Weber fraction or *w*). For children in the Easy-First condition, we found that the *w* value that resulted in the best fit of the model was 0.20 (r^2 = .81); for children in the Hard-First condition, *w* was 0.41 (r^2 = .63); and for children in the Random Order condition, *w* was 0.33 (r^2 = .81) (Fig. 3). Because lower values for *w* indicate better precision, this means that children in the Easy-First condition showed better ANS precision than children in the Random Order condition and the Hard-First condition. In fact, children in the Easy-First condition tended to have better *w* values than previous estimates for typically developing 5-year-olds (Halberda & Feigenson, 2008; Odic, Libertus, Feigenson, & Halberda, 2013; Piazza et al., 2010), another suggestion that experiencing the Easy-First ANS manipulation improved children's ANS precision. In contrast, children in the Hard-First condition showed worse ANS precision (i.e., higher *w* values) than children in the Random Order condition. In fact, children in the Hard-First condition had *w* values that were roughly equivalent to those observed for typically developing 9-month-old infants (Lipton & Spelke, 2003)—a suggestion that the brief Hard-First ANS manipulation negatively impacted children's ANS precision.

We next asked how children in the Random Order condition of the Supplemental Experiment performed on the Symbolic Math Transfer Task, as compared with children in the Easy-First and Hard-First conditions of Experiment 1. A one-way ANOVA with Symbolic Math Transfer Task accuracy as a dependent variable and ANS manipulation condition as factors revealed a significant main effect of condition, $F(2, 27) = 3.89$, $p = .033$, $\eta_p^2 = .22$ (Fig. 4). Children in the Easy-First condition performed better on the Symbolic Math Transfer Task (82.78%, *SD* = 14.21) than children in the Hard-First condition (60.56%, *SD* = 24.21), and children in the Random Order condition performed at an intermediate level (75.00%, *SD* = 13.92) (Fig. 4). Tukey's HSD post hoc analyses revealed a significant difference between the Easy-First and Hard-First conditions ($p < .05$) but no significant difference between the Easy-First and Random Order conditions or between the Hard-First and Random Order conditions. Given the relatively small sample sizes we used here, the lack of significant difference between performance in the Random Order condition and the other two conditions may have been a matter of power. However, these results start to suggest that, similar to the results on the ANS Manipulation Task, children's performance on the Symbolic Math Transfer Task may have been boosted by the Easy-First ANS manipulation and impaired by the Hard-First ANS manipulation, with performance in the Random Order condition falling in the middle.

We next examined children's performance on the Symbolic Math Transfer Task in greater detail. This task included four different item categories from the TEMA-3 (numbering, number comparison, calculation, and numeral literacy), allowing us to ask whether ANS precision was linked to the specific math abilities captured by these items. First, we averaged children's performance on the items in each category to calculate each child's percentage correct for each math category. Next, to ask whether ANS confidence hysteresis affected children's performance in all or just some of these math categories, we analyzed performance in a 4 (Math Category: numbering, number comparison, calculation, or numeral

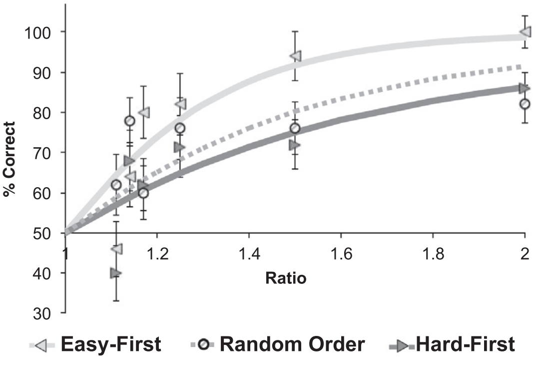

Fig. 3. Children's ANS performance as a function of ratio and ANS manipulation condition (Easy-First, Random Order, or Hard-First). The curves are the group fits for the standard psychophysical model that has a single free parameter—the Weber fraction (*w*). Error bars indicate ±1 standard error.

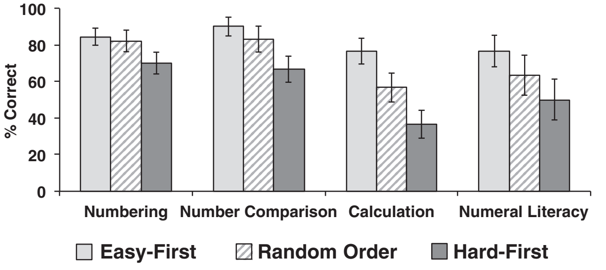

Fig. 4. Children's performance on each question category of the Symbolic Math Transfer Task, separated by ANS manipulation condition (Easy-First, Random Order, or Hard-First). Error bars indicate ±1 standard error.

literacy) × 3 (Condition: Easy-First, Hard-First, or Random Order) repeated measures ANOVA with percentage correct as the dependent measure. This revealed a main effect of math category, $F(3, 81) = 8.97$, $p = .001$, $\eta_p^2 = .25$. Across all children, average performance for numbering was 78.89% ($SD = 19.2$), for number comparison was 80.00% ($SD = 24.13$), for calculation was 56.67% ($SD = 29.23$), and for numeral literacy was 63.33% ($SD = 35.40$), suggesting that calculation and numeral literacy abilities are harder or emerge later than numbering and number comparison (consistent with previous research, e.g., Ginsburg & Baroody, 2003). We also found a significant effect of condition, $F(2, 27) = 5.06$, $p = .014$, $\eta_p^2 = .27$, consistent with our earlier finding of poorer performance for Hard-First children compared with Easy-First children. Importantly, there was no Condition × Math Category interaction, $F(6, 81) = 0.69$, $p = .58$, $\eta_p^2 = .05$, suggesting that manipulating children's ANS precision equally affected their symbolic math performance across all of the math categories tested (Fig. 4).

Discussion

The results from the Supplemental Experiment help to place the findings from Experiment 1 in a clearer light. Children in the Random Order ANS Manipulation condition (Supplemental Experiment) exhibited performance that was intermediate between that of children in the Easy-First and Hard-First Manipulation conditions (Experiment 1) on both the ANS Manipulation Task and the Symbolic Math Transfer Task. This suggests that ANS confidence hysteresis can both enhance and disrupt the precision of ANS representations and that these effects transfer to symbolic math performance. This is so despite the fact that all children, regardless of ANS manipulation condition, completed identical Symbolic Math Transfer Task items, identically ordered. Furthermore, the analyses performed in the Supplemental Experiment suggest that ANS confidence hysteresis likely affected children's performance across a range of symbolic math skills. However, it is important to note that we do not expect that the ANS confidence hysteresis effect would transfer equally across all aspects of formal math abilities (e.g., it might not affect memorized math facts). In this first inquiry, we chose to focus on those math abilities that have previously been shown to correlate with children's ANS precision (Libertus et al., 2013a), but determining the scope of the causal relations from ANS precision to symbolic math performance (in terms of which specific abilities are affected and at which ages) will be important to explore in the future.

General discussion

Several reports have shown a link between the intuitive ANS and symbolic mathematics abilities, but to date no study has shown a causal link between ANS *precision* and math. Here we investigated whether ANS precision causally contributes to individual differences in symbolic math performance during early childhood. We gave 5-year-olds a brief manipulation task involving non-symbolic numerical discriminations that progressed either from easy discriminations to hard ones or from hard ones to easy ones or where difficulty varied randomly across trials. Immediately afterward, we tested children's math or verbal performance.

Our experiments reveal three main findings. First, we replicated the effect of ANS confidence hysteresis (Odic et al., 2014). The order in which children performed ANS discriminations, even in a brief task, significantly affected their observed ANS precision. Children who completed a series of ANS discriminations that progressed from easy ratios to hard ones showed significantly better ANS precision than children who completed the same discriminations in the reverse order. This result extends the emerging body of research on the malleability of ANS precision, suggesting that accuracy in making approximate numerical judgments is sensitive to experimental manipulations such as computer-assisted interventions (Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006), order of trial difficulty (Odic et al., 2014), and the presence of feedback (DeWind & Brannon, 2012).

The finding that ANS precision differed between two training groups raises the important issue of the extent to which the precision of ANS representations is stable versus malleable. On the one hand, longitudinal investigations show that individual differences in ANS precision at one developmental time-point predict future ANS precision (Libertus & Brannon, 2010; Libertus et al., 2013a; Starr et al., 2013) and retrospectively correlate with school math performance over a period of months or years (Halberda et al., 2008). Such findings suggest some stability in numerical approximation abilities. We might think of this as a "trait" of ANS precision, where individual differences in this trait can be measured as early as infancy and hold predictive power across development. At the same time, how one performs in a particular task on a particular day depends on various factors, including experience (e.g., task-specific training; DeWind & Brannon, 2012; Park & Brannon, 2013) and motivation (Lindskog, Winman, & Juslin, 2013). These temporary performance changes evoke the notion of a "state" of ANS precision rather than an unchanging trait. Similar notions of trait and state have been important throughout psychology. As an analogy, consider the example of executive functions—for which people have been shown to differ in stable ways across development (e.g., Harms, Zayas, Meltzoff, & Carlson, 2014; Miyake & Friedman, 2012) but for which specific experience can either enhance abilities (e.g., Best, 2012; Diamond & Lee, 2011) or diminish abilities (e.g., Quinn & Joormann, 2015). Similarly, we suggest that ANS precision has both an enduring stable component (responsible for many of the previously reported correlations with symbolic math ability) and a transitory state that is responsive to experience (as in the confidence hysteresis effect demonstrated here as well as other reports of ANS training).

However, like the findings by Odic and colleagues (2014), our results do not answer the question of how trial order affects the "state" of ANS precision. Although characterizing the mechanism underlying ANS confidence hysteresis is beyond the scope of the current work, it is clear that this is an important direction for future inquiry. One possibility is that instead of influencing the underlying ANS representations themselves, feedback may have increased participants' task motivation, which in turn could boost ANS performance (Lindskog et al., 2013). The finding that ANS hysteresis is not observed when children in the Easy-First trial order receive no feedback at all (Odic et al., 2014) is perhaps consistent with this. However, Easy-First children show better ANS precision even when given "reversed feedback" in which correct answers receive negative responses and vice versa (Odic et al., 2014). This suggests that changes in ANS precision are unlikely to be driven by a diffuse motivational boost that results from external reward. A different but potentially related possibility is that experiencing difficult trials at the start of the task caused children in the Hard-First condition to give up in their attempt to numerically compare the stimulus arrays and to instead guess randomly on the hardest trials. In contrast, children in the Easy-First condition may have persisted in attempting to perform the more difficult discriminations because of their successes on previous trials. These possibilities are ripe for investigation.

Our second key finding concerns the relationship between the ANS and symbolic math ability. Previous findings showed a correlation between the precision of people's approximate number representations and their performance on standard math assessments involving numerals, equations, and word problems (e.g., Halberda et al., 2008). Here we used ANS confidence hysteresis to test the causal nature of this relationship. We found that when we induced a temporary change in children's observed ANS precision, this also changed children's performance on a symbolic math task. Children performed significantly better on the math task after having made a series of ANS judgments in an Easy-First trial order, compared with children who made the very same ANS judgments in a Hard-First trial order. This extends our understanding of the relationship between the ANS and symbolic math in several ways. First, it is the first demonstration of a causal link between ANS *precision* and symbolic math performance, in contrast to previous studies that found a link between ANS training tasks and math but without improvements in precision (Hyde et al., 2014; Park & Brannon, 2013, 2014). Second, we found evidence of ANS training even when nearly all aspects of ANS experience were equated across groups. All of the children completed identical ANS discriminations, thereby controlling for the total amount of numerical practice, the types of numerical computations performed, and the ratio-defined difficulty of the individual trials (in contrast to previous studies where training experience differed qualitatively between groups; Hyde et al., 2014; Park & Brannon, 2013, 2014). In our task, the only difference between children's training was the order in which children completed the ANS trials. Finally, we found that the training-induced improvement in symbolic math performance was not restricted to basic arithmetic (e.g., as in the studies by Hyde et al., 2014, and Park & Brannon, 2013, 2014). Rather, we observed changes in a range of symbolic math abilities, including solving word problems, counting, and reading and writing numerals. Impressively, this transfer from ANS confidence hysteresis to symbolic math was observed after children had completed only approximately 5 min of ANS discriminations.

The third key finding from our work was that the effects of ANS confidence hysteresis did not transfer to performance on a non-numerical vocabulary task in spite of the math task and the vocabulary task being matched in overall difficulty. This suggests that the changes we observed were not due to ANS confidence hysteresis affecting general motivation or mood. In fact, if changes to motivation or mood were the critical factors, we might have predicted a pattern opposite to that observed. Children in the Easy-First condition experienced the hardest numerical discrimination trials (on which they received the most negative feedback) at the very end of the ANS manipulation, immediately before the transfer task, whereas children in the Hard-First condition experienced the opposite. If carryover of motivation or mood were most strongly influenced by the trials immediately preceding the transfer task, this would predict better symbolic math performance for children in the Hard-First condition than for children in the Easy-First condition. Instead, we saw that children who completed the hardest ANS discrimination trials right before the math task (i.e., children in the Easy-First condition) performed best. Our results, therefore, suggest that manipulation of ANS precision alters symbolic math performance in a way that may be relatively independent of some of the socioemotional factors that have been previously identified to play a role in children's performance, including motivational goals (Dweck, 1986) and emotional state (Bryan & Bryan, 1991), although we believe that these factors may play an important role in children's math performance as well.

Of course, a key question raised by our results is how manipulating children's ANS precision alters math performance. At the moment, we can only speculate about possible mechanisms and outline future experiments that may help to answer this question. One possibility is that the ANS plays a role in supporting children's intuitions about math problems, even those presented in symbolic format. If the ANS is used to generate ballpark estimates for correct answers to symbolic problems, enhancing ANS precision could in turn increase children's accuracy and efficiency at solving symbolic problems (Greeno, 1991; Jordan, Glutting, & Ramineni, 2010). Future research could test this possibility by investigating the influence of ANS confidence hysteresis on children's error patterns in symbolic math problems. Another possibility is that ANS precision plays no direct or supporting role as children attempt to solve math problems but that, rather, children experience a sense of self-efficacy that is specific to the mathematical domain (Hackett & Betz, 1989; Vo, Li, Kornell, Pouget, & Cantlon, 2014). That is, ANS confidence hysteresis might affect children's internal sense of their ability to reason about quantities. For example, children who experienced the Easy-First ANS manipulation might have been more likely to think of themselves as mathematically competent, thereby leading to more perseverance in subsequent symbolic math problems. This is an intriguing possibility, and further research could use ANS confidence hysteresis as a tool to explore the emergence of math attitudes and their relation to math performance. Confidence hysteresis could also be used as a tool to ask whether changes in precision in other, non-numerical representations influence children's symbolic math abilities.

Finally, an interesting question for future research concerns the potential use of ANS confidence hysteresis as a practical intervention method for improving children's math abilities. We believe that the feasibility of such an intervention depends greatly on the robustness and duration of the confidence hysteresis effect. Replications of the current finding in different populations (including participants of a wider range of ages than those tested here) and in more diverse settings (including classrooms) are needed to bear on this. Here we used ANS confidence hysteresis as only a brief manipulation—not a full intervention. Further exploration of the duration and mechanisms of ANS confidence hysteresis effect will help to extend the current findings to practical applications.

In summary, we asked whether precision of the ANS causally affects children's mathematical performance. We demonstrated that a brief change in 5-year-olds' nonverbal numerical discrimination performance, induced by ANS confidence hysteresis, transferred to children's performance on a symbolic math task but not to their performance on a non-numerical verbal task. These findings suggest a causal link from ANS precision to symbolic math performance.

Acknowledgments

We thank all of the children and parents for their participation in the study. This research was conducted at Johns Hopkins University and was funded by National Institutes of Health – United States Grant R01 HD057258.

Appendix A

Numbers of dots presented in the Easy-First ANS manipulation marked by trial number and side of presentation.

Trial number	Yellow (left)	Blue (right)
1	10	5
2	6	9
3	9	6
4	10	5
5	5	10
6	6	9
7	10	8
8	10	5
9	9	6
10	5	10
11	10	8
12	6	9
13	10	8
14	12	14
15	10	8
16	7	8
17	14	12
18	7	8
19	8	10
20	12	14
21	8	7
22	7	8
23	9	10
24	12	14
25	9	10

(*continued on next page*)

Appendix A (*continued*)

Trial number	Yellow (left)	Blue (right)
26	10	9
27	14	12
28	10	9
29	8	7
30	10	9

Appendix B

Symbolic Math Transfer Task items marked according to original item number in the TEMA-3.

10. Give me _ tokens
11. Hold up _ fingers
13. What number comes next; __, and then comes?
15. Write the number
16. How many does he have altogether?
19. Which is more, . . .?
20. Which is more, . . .?
21. Count up as high as you can. (stop at 21 or 42)
22. What number comes next; __, and then comes?
23. Count these dots with your finger
24. Count backward, start from 10
26. How much are __ and __ altogether?
27. Which is closer to __, __ or __?
29. What number is this?
32. How much is __ and __ altogether? (must count up from larger addend)
35. What number is this?
38. Count these dots with your finger

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